Measuring Idiosyncratic Risks in Leveraged Buyout Transactions

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Abstract:

We use a contingent claims analysis model to calculate the idiosyncratic risks in Leveraged Buyout transactions. A decisive feature of the model is the consideration of amortization. From the model, asset value volatility and equity value volatility can be derived via a numerical procedure. For a sample of 40 Leveraged Buyout transactions we determine the necessary model parameters and calculate the implied idiosyncratic risks. We verify the expected model sensitivities by varying the input parameters. For the first time, we are able to calculate Sharpe Ratios for individual Leveraged Buyouts, thereby fully incorporating the leverage risks.

JEL classifications: G13, G24, G32

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1. Introduction

Leveraged Buyouts (LBOs) are transactions in which a financial investor takes over a company via a special purpose vehicle. The funding of the special purpose vehicle is typically composed of several layers of debt and non-traded equity claims. In most of the cases the debt/equity ratios of LBO transactions are above what is considered normal. These two properties - the illiquidity of the private equity market and the leverage ratio make LBOs high-risk investments. No adequate methods have been developed so far to successfully determine the risks implied by particular transactions; consequently, risk-adjusted performance ratios that benchmark individual transactions are non-existent. In this paper we use contingent claims analysis (CCA) to provide a measure of inherent risks in LBO transactions. Typical LBO deal structures mean, however, that standard CCA models cannot be used, and so adjustments need to be made.

Green (1984), Sick (1989), and Arzac (1996) point out that when debt/equity ratios in LBO transactions are high, the similarity of the equity valuation to a call option valuation becomes obvious. Shareholders can exercise the option, take over the company and redeem the debt if the enterprise value is above the liabilities. Default is triggered if the company value reaches the level of liabilities. Leaving aside agency costs, debt financing thus induces more speculative behavior of the shareholders, according to Myers (1977), because they have unlimited earnings potential but only limited risk. This makes a CCA-approach a promising method for estimating the implied volatilities of transactions in which shareholders and lenders bear a certain amount of idiosyncratic risk.

Underlying the CCA approach is the assumption, that the arrangers of a transaction will model future company development and potential scenarios. In accordance with their models, they agree a purchase price with the seller and structure the transaction with several equity and debt layers. At this moment, prices for the different equity and debt strips are fixed. The LBO

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sponsors have a finite investment horizon and plan to sell their stakes after a certain holding period. The holding period, which is reflected in their transaction model, determines a sponsor's expected internal rate of return on the investment. The transaction model is based on free cash flow estimates and considers due redemption of the debt and adherence to the debt covenants. Lenders usually ask for an immediate reduction of their exposure in large scales after the closing of a LBO transaction. Hence, an appropriate leverage ratio must be applied that secures the desired return on equity and that fulfills the lender requirements regarding redemption and covenants. In a simplified model, where there is just a single layer of common equity and a single layer of risky debt with one amortization payment, the CCA approach determines both, the transaction's and the equity's idiosyncratic risks. These risks depend on the deployed capital structure at closing, the planned investment horizon, the assumed debt redemption capabilities of the target company, the risk-free interest rate, and the debt-credit spread. In other words, for a given transaction, the arrangers assume a certain volatility in the target company's asset and equity values when they agree on debt and equity prices, bearing in mind the amortization schedule over the investment horizon – which is the moment of closing the transaction.

To a manually collected sample of 40 LBO transactions we apply the Ho and Singer (1984) model in order to price risky corporate debt (with one amortization payment). We implement a numerical approach to calculating the implied asset value volatilities, which range from 13.6% to 106.4% p.a. (mean 35.3% and median 30.4% p.a.). From the asset value volatilities we derive the idiosyncratic equity risks for the same transactions, obtaining high values ranging from 57.8% to 182.5% p.a. (mean 94.1% and median 93.5% p.a.). This is the first time that idiosyncratic risks are calculated for individual LBO transactions. Using data on the returns received by the LBO sponsors, we can – also for the first time - calculate Sharpe ratios for the individual LBO transactions.

It is not the purpose of this paper to present a mean idiosyncratic risk for the asset class, but to propose an approach to calculate risk-adjusted returns, bearing in mind different degrees

of leverage in the transactions. The model can be applied to the benchmarking of current and future transactions, which will lead to improved understanding of, and transparency in, the asset class. Comprehensive analyses of the mean risk-adjusted return of the asset class will be possible with the presented approach if more data on individual transactions becomes available for academic research.

2. Literature Overview

2.1. Literature on Financial Risk Measures for the Venture Capital and Leveraged Buyout Market Segment

Several papers deal with associated risks in non-quoted equity markets, but do not usually differentiate between different market segments, such as venture capital (VC) and LBOs, and do not pay sufficient attention to their particularities, especially when referring to LBO transactions. For instance, Cochrane (2005) reports a mean volatility of 86% p.a. for a sample of 16,638 private equity transactions – calculated via maximum likelihood estimates and sample bias correction for unobservable returns but does not differentiate between VC and LBOs; more importantly, he does take account of the degrees of leverage deployed in the LBOs. Although Kaplan and Schoar (2005) analyze the performance of private equity investments and create a sub sample of LBOs, rather than consider idiosyncratic risk, they consider systematic risk - which they assume to be both equal for every transaction, and equal to the systematic risk of the S&P 500 Index. Their approach also implies, therefore, that leverage in public and private markets is identical. Quigley and Woodward (2003) create a VC index similar to that of Peng (2001), and report a mean annual standard deviation of 14.6% of the asset class, while Peng (2001) reports annual standard deviations between 9.5% and 70.3% for the period from 1987 to 1999. Both papers focus on correcting missing values and selection bias and fail to either create a LBO sub-sample or to consider individual LBO capital structures in their approach. Ljungqvist

and Richardson (2003) distinguish between VC and LBO market segments and analyze LBO performance while controlling for systematic risk. However, because they do not have access to exact data for individual deals, they assume industry averages for the debt/equity in their calculations of LBO beta factors. These authors report an average beta factor of 1.08 for their LBO sample. Groh and Gottschalg (2008) investigate LBO performance and focus on systematic risk for transactions using detailed information on debt/equity ratios. For several scenarios based on differentiated assumptions about the risk of debt and of debt tax shields, they calculate average equity beta factors ranging from 0.78 to 2.57 at transaction closing. However, it is not clear which scenario is the "right" one. Furthermore, their focus on systematic risk does not enable individual transactions or undiversified portfolios to be benchmarked.

This paper differs from the existing literature regarding two important aspects. First of all, we focus on idiosyncratic risk and exclusively on LBO transactions. Secondly, our proposed model is derived from CCA, adapted for the characteristics of LBOs and transferred to the asset class. This represents a unique and promising approach to the calculation of implied transaction risks that account for the risk superimposed by the debt deployed to financing the transaction. This approach will enable comprehensive risk-adjusted performance analyses to be performed when more data (e.g. on current and future transactions) become available.

The next section describes the origin of our CCA model and related literature.

2.2. The Origin of our CCA Model

Black and Scholes (1972) and Merton (1973 and 1974) devise models to calculate the equity value of a company, given the value - and fluctuations in value – of its assets, and including the notion of a pure discount bond, time to maturity of the bond, and the risk-free interest rate. Assuming that all the other parameters are given, the models can be solved numerically for the asset value fluctuation. This asset value fluctuation is called implied volatility, and represents a measure of the expected fluctuation in company value. It can be assumed that

originators of a LBO transaction set a price for equity according to that risk. All the other parameters are usually known on closing a LBO transaction and the Black and Scholes approach can be directly extended to calculating the implied volatility of a LBO if this is financed with default-free zero bonds.

However, since LBO transactions are typically characterized by a large proportion of debt at closing and substantial subsequent debt redemptions, decreasing debt levels have to be taken into account in the CCA valuation model. Furthermore, some deployed debt instruments allow flexible amortization payments. This complicates the CCA approach because the exercise price of the option is uncertain, as discussed in Fisher (1978). Moreover, standard boundary conditions are no longer valid and interest due dates convert the approach into a compound option, namely, a path-dependent valuation problem. Black and Cox (1976), and Ho and Singer (1982) expand the Merton (1974) pricing model and introduce several bond indenture provisions, such as safety covenants, subordination arrangements, and restrictions on the financing of interest and dividend payments.

Geske (1977 and 1979), and Brockman and Turtle (2003) deal with the path-dependent option problem. Unfortunately, the models cited are not suitable for our purposes, both because they include many parameters that are not observable in our sample data and because they require assumptions which cannot be taken as given in LBO transactions. The Geske (1977) approach, for example, requires equal sinking fund payments for each coupon date in order to completely retire the face value of the debt by maturity. In LBO transactions debt is not usually fully redeemed within the financial investor's planned holding period.

Jones, Mason, and Rosenfeld (1983 and 1984) formulate a partial differential equation and boundary conditions that price risky bonds with sinking fund provisions. They empirically test their model and conclude that their approach is more appropriate than a naive model based on the assumption of risk-less debt, especially when valuing low grade bonds issued by companies with high debt/equity ratios.

Based on the Cox-Ross-Rubinstein (1979) binomial model, Ho and Singer (1984) derive a closed form solution to pricing a risky coupon bond with a single redemption payment either made by open market repurchase of the bond or by calling the bond at par value. In LBO transactions debt is usually redeemed according to an agreed schedule. Payments are made at discrete points and typically in different amounts. The Ho and Singer (1984) model does not reflect different kinds of amortization schedules, but reduces the complexity by assuming a single payment in the lifetime of the bond. However, this model perfectly matches one important typical feature of LBOs, by allowing for a bullet payment to redeem the outstanding principal at maturity. We adapt this model to LBO transactions and, from the observable price of the debt deployed in the transactions, conclude the implied volatility of the target company's asset value. From the asset value volatility we can derive the subsequent implied equity risk.

We describe the Ho and Singer (1984) model and the assumptions necessary to be able to transfer it to our LBO sample in the next section.

3.   The LBO CCA model

The approach refers to Black and Scholes (1973), who emphasize that common stock of a corporation that has outstanding coupon bonds can be considered as a compound option. On each coupon date, shareholders have the option of buying the next option by paying the coupon, or of forfeiting the firm to the bondholders. Their final option is to repurchase lender claims by paying off the principal at maturity. If bond indentures require amortizations the shareholders have the additional options of either buying the next option at the redemption date or of forfeiting the company. To our knowledge, the literature does not provide a closed form solution for valuing risky coupon debt redeemed at discrete points in time (possibly in different amounts) for a non-zero principal at maturity. However, the Ho and Singer (1984) model comes close to that desired feature. The authors provide a closed form solution for a discount bond with one

amortization payment and a final payoff at maturity. The modifications necessary for transfer of this model to typical LBO transactions are as follows:[1]

1. The value of the firm's assets is independent of its capital structure.

2. The firm's capital structure consists of a single equity and a single debt layer.

3. The yield curve is flat and non-stochastic.

4. Until the maturity of the debt, the firm's investment decisions are known.

5. The firm does not pay dividends and does not make any other distributions to shareholders.

6. Default occurs when the firm fails to satisfy the bond indentures. If defaulted, the shareholders forfeit the assets to the lenders costlessly.

7. The amortization payment is financed with new equity.

8. The amortization payment is fixed in the bond indentures as a proportion of the debt outstanding at a given time.

We employ the following notation:

$V(t)$ is the value of the firm at t, when the amortization payment falls due.

F is the value of outstanding debt at the exit date of the LBO transaction.

sF is the value of debt to be amortized as a proportion of the value of outstanding debt at the exit date of the LBO transaction.

T is the holding period of the LBO transaction.

τ is the time from the amortization payment to the exit date, i.e. $\tau = T - t$.

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[1] Compare to Ho and Singer (1984) p. 317.

B[V(t);F,τ] is the value of the debt remaining after the amortization, i.e. the value of a discount

bond with future Value F and time to maturity τ.

c   is the constant yield of the debt over the holding period.

$sFe^{-c\tau}$ is the amortization payment.

According to Merton (1974), the value of the company's equity just after the amortization

payment can be expressed as the value of a call option on the underlying value of the firm, with

exercise price F and time to expiration τ. Default is triggered when the asst value becomes

smaller than the amortization payment. It is further assumed that trading can take place

continuously and that the value of the company's assets follows an Itô diffusion process

$dV / V = \alpha\, dt + \sigma\, dw$, where α is the constant instantaneous expected growth of the asset value

of the firm, σ is the instantaneous standard deviation, and dw is the increment of a standard

Gauss-Wiener process. The standard deviation of the underlying assets σ is the parameter in

question, which is finally solved numerically. Ho and Singer (1984) formulate a solution that can

be transferred for valuing the debt of a company on closing a LBO transaction, using our

notation and the necessary assumptions:

$$D = VN\left(x_b - \sigma\sqrt{t}\right) + e^{-rt}\int_{x_b}^{\infty} f(x)B(x)dx + N\left(-x_b\right)sFe^{-c\tau-rt}, \tag{1}$$

where x_b is the solution of

$$x_b = \frac{\ln\dfrac{sFe^{-c\tau} + B(x_b)}{Ve^{rt}}}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}, \tag{2}$$

with

$$B(x) = N\left(y - \sigma\sqrt{t}\right)Ve^{-\frac{1}{2}\sigma^2 t + \sigma\sqrt{t}x + rt} + N\left(-y\right)Fe^{-r\tau} \tag{3}$$

and

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$$y = \frac{\ln\dfrac{Fe^{-rT}}{V} + \dfrac{1}{2}\sigma^2 T - x\sigma\sqrt{t}}{\sigma\sqrt{\tau}}.$$ (4)

The standard normal density function f(s), N(z) is the standard normal cumulative distribution function, and x_b is the bankruptcy point at t. The equations are equivalent to the Geske (1977) compound option formula for pricing a serial bond. The value of the debt is the present value of three claims associated with the debt issue, as follows: the first term is the present value of the lender's claim if the company defaults; the second term is the present value of the unredeemed debt if the firm satisfies the amortization payment; and the last term is the present value of the redeemed debt if the firm satisfies the amortization requirement. The partial derivatives of equation (1) are as follows:

$$D_V = N\!\left(x_b - \sigma\sqrt{t}\right) + e^{-rt}\int_{x_b}^{\infty} f(x)\frac{\partial B(x)}{\partial V}dx > 0$$

$$D_F = e^{-rt}\left[\int_{x_b}^{\infty} f(x)\frac{\partial B(x)}{\partial F}dx + N(-x_b)se^{-c\tau}\right] > 0$$

$$D_r = -TFe^{-rT}\int_{x_b}^{\infty} f(x)N(-y)dx < 0$$

$$D_T = -rFe^{-rT}\int_{x_b}^{\infty} f(x)N(-y)dx < 0$$

$$D_s = Fe^{-c\tau-rt}N(-x_b) > 0$$

$$D_c = -\tau N(-x_b)sFe^{-c\tau-tr} < 0$$

The partial derivatives prove the expected influence of V, F, T, c, and r. However, an increase in the redemption ratio s increases the expected redemption payments and, hence, also increases the debt value.

For the calculation of σ we implement a Brent-Algorithm.[2] This avoids the calculation of derivatives required for the Newton-Raphson method used by other researchers as Brennan and Schwartz (1977), Chiras and Manester (1978), Latané and Rendelman (1976), and Schmalensee and Trippi (1978). Indeed, the Brent-Algorithm (see appendix) converges more slowly but is also more stable.

4.   The Data Sample and Necessary Simplifications

From a collection of 122 private placement memoranda (obtained from institutional investors) that include 1,001 exited LBO transactions conducted in the USA between 1981 and 2004, the minimal required information was obtained for only 133 transactions. This includes holding period, purchase price at closing, capital structure at closing, and company valuation and capital structure at exit. Furthermore, to perfectly qualify for our study, the target companies' capital structures ought to have consisted of a single equity claim and a single debt claim. Unfortunately, this was not the case in any of the transactions. As recommended by Jensen and Meckling (1976), investors usually apply a complex mix of equity and debt claims to minimizing agency costs. Furthermore, as underlined by Jensen (1986), the burden of debt should lead to more efficient organizations, spur managers to invest in value increasing projects only, and increase free cash flows and, hence, company valuations. Consequently, LBOs are usually structured with several different layers of debt and equity. The principle of strip financing - as comprehensively discussed in Jensen (1989) and Harris and Raviv (1989), and empirically investigated by Kaplan and Stein (1993) - plays a major role in structuring transactions. Financial sponsors often employ conversion and option rights to secure an alignment of interests between different financing parties, managers, and employees of the LBO corporation. There is no limit regarding the flexibility of the design of the structured claims. Debt might be convertible to equity and equity to debt. Payment-in-kind debt-instruments are often deployed that allow flexible

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[2] See Brent (1973).

redemption. Beside senior debt securities with relatively low interest rates but precisely defined amortization schedules, junior and mezzanine strips with higher yields also allow for flexible payments. All of these facets are well represented in our data sample, but unfortunately not well documented. However, we can not cover this flexibility with our CCA model, and so we explain below how we treat complex claims and capital structures in our sample.

Since the CCA model prohibits additional financing rounds – of either equity or debt – many transactions have to be discarded from the sample. Additional financing rounds are not an issue *per se* that prevent the applicability of our model on current or future transactions. If additional financing rounds are planned *ex ante* by the buyout sponsors they can be considered in the initial financing scheme and therefore, also in our model. However, we do not observe sufficient information on these additional rounds to correct for them, and hence, discard those transactions from the sample. Furthermore, the model does not capture option or conversion rights, if structured. To cover additional contingent claims a more complex approach had been needed to be derived. However, for most of the above-mentioned 133 sample transactions, no detailed information about the design of the particular claims is available, and so it is still impossible to test a more sophisticated model. Only a few of the cases provide details of the proportions of common and preferred equity or of contractual loan obligations. In these cases we were finally able to make a decision about the fit to the proposed CCA method. Consequently, transactions that violate the necessary model requirements regarding the deployed capital structure and/or additional financing rounds are discarded from the sample if the transaction was explicitly structured with convertible debt or other capital layers providing contingent claims valuing more than 5% of the company's total asset value at closing, and if a company's equity was structured as preferred and common and if there was a dividend guarantee or similar for more than 10% of the equity stakes. For all the transactions we assume that no dividends were paid and that different equity layers can be regarded as equal. For all these other transactions, we also assume that the different debt layers - even if there is a seniority ranking among them -

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can be represented by just one term loan, under the contractual obligation for redemption to a certain level at the halfway point to maturity. Time to maturity is the holding period of the LBO sponsors. Furthermore, as the proposed CCA model does not allow additional external financing, all transactions in which the final debt levels exceed the compounded initial debt levels are discarded, i.e. in which

$$D > Fe^{-cT}.$$

This leaves just 40 LBO transactions in our sample that qualify for the proposed method for quantifying implied transaction risks. Table 1 summarizes the descriptive statistics of our sample.

Insert table 1 here

The first and last transactions were closed, respectively, in April 1987 and March 2003. The mean closing date is October 1996, while the median is June 1997. The exit dates range between November 1993 and December 2003, with a mean of February 2000 and a median of June 1999. The holding periods of the transactions range between 3 months and 9 years and 11 months. The mean holding period is 3 years and 4 months, while the median is 2 years and 5 months. This indicates that most transactions in our sample have a rather short holding period. The enterprise values of the target companies range between $3.6 m and $1,679.7 m, with a mean of $181.7 m and a median of $95.9 m. At the exit date, asset values range between $10.5 m and $1,347.5 m, with a mean of $227.5 m and a median of $131.5 m. Regarding the leverage ratios for our transactions - at closing between 0.3 and 8.0, with a mean of 2.8 and a median of 2.5 - we observe that many of the LBOs are, in fact, very highly leveraged. However, the decrease in the leverage ratio over the holding period - to a minimum of 0 and a maximum of 3.1, with a mean of 0.7 and a median of 0.4 - reveals that debt redemption is an important

feature of LBO transactions. Referring to the short holding periods, the high deleverage speed is of note. Transactions sponsors invested between $0.4 m and $248.4 m, and mean and median exposures, respectively, were $47.8 m and $25.0 m. The sponsors benefited from their exposure by receiving payoffs between $8.0 m and $631.2 m, with a mean of $140.8 m and a median of $95.2 m. They thus obtained internal rates of return on their cash flows of a minimum of -39.6% p.a. and a maximum of 267.5% p.a. with a mean of 77.6% p.a. and a median of 57.9% p.a. The large standard deviations reflect the large fluctuations in the parameters in the sample and, hence, reveal the diversity of the transactions.

This sample is not a random draw from the population of US LBO transactions for several reasons. Firstly, our data is gathered from private placement memoranda, which are typically edited by buyout firms for the purpose of developing a marketing instrument for generating a new fund. Since only successful players will be able to raise another fund, the sample is biased to more successful transactions. Secondly, we do not have private placement memoranda of all US buyout firms undergoing a fundraising process and we are not able to determine any parameter that influences the availability of a memorandum. Thirdly, inclusion in the sample depends on the information available on the transaction, which is arbitrarily edited in each private placement memorandum and for each transaction. Finally, inclusion in the sample further depends on our own selection process described above.

The mean internal rate of return yielded by the invested equity also reveals that the sample is biased towards successful transactions. However, it is not the purpose of this paper to benchmark one asset class against others, but to propose a method for calculating risk-adjusted returns. Hence, sample bias is not an important issue in terms of the results of this study.

5. Analysis and Results

To apply the proposed model to our sample we need to specify further parameters including the debt interest rate, the amortization schedule (timing and amounts to be redeemed),

and debt maturity. Unfortunately, the required information is not available in the documentation for our sample LBOs. However, the risk free interest rate can be calculated as the geometric mean of the one-year US Treasury rates for the periods corresponding to the individual LBO holding periods. Regarding the debt interest rate, we simplify and use two different approaches; in the first, we define debt yield to maturity as 8.0% p.a. for all transactions, and in the second (part of the sensitivity analysis), we apply a constant credit spread of 3% p.a. on the risk-free rate for each transaction. Both approaches disregard different degrees of credit risk for the individual transactions and changing yield curves over the time. However, since it is not the purpose of this paper to precisely calculate the idiosyncratic risks of the historic transactions, but rather to propose an approach for benchmarking current and future ones (in which appropriate interest rates can be considered), our simplification seems acceptable.

Furthermore, detailed information on debt amortization schedules is usually also not available. However, since the Ho and Singer (1984) model requires a single amortization payment, we assume that this payment is made in the middle of the holding period of the individual transactions. The amount of the payment is then given by sFe^{-ct}, while

$$s = \frac{D}{F}e^{cT} - 1,$$

when the amortization payment is made in the middle of the holding period with $t=\tau$.

Finally, we assume debt maturity to be equal to the holding period of the transaction. This corresponds with the view that the observed LBO transactions in our sample exactly match the scenario expected by the sponsors. Although this is an unlikely situation, it enables us to prove that our concept is transferable to current and future transactions. Transfer is possible because, for current and future LBOs, the sponsors create a transaction model and all the necessary data can be taken as computed from that model. Since the prices of all equity and debt claims are determined by the forecast model, it is evident that these prices reflect the expected holding

period, expected amortizations, debt yield, outstanding debt at the end of the holding period, and, consequently, the implied transaction and equity risk according to the CCA approach.

Using the numerical Brent algorithm (Brent 1973), and the data and definitions given above, we calculate the implied idiosyncratic risks for our sample of LBO transactions. The parameters and results for all 40 transactions are described in table 2.

Insert table 2 here

The lowest implied volatility of the company asset values is 16.7% p.a. while the highest variation is 103.2% p.a. The mean of the implied company valuation fluctuations for all 40 transactions is 37.8%, the median is 33.4%, and the standard deviation is 19.0%.

To benchmark the returns yielded by the individual transactions we derive the volatility of the invested equity from the asset value fluctuations. Black and Scholes (1973) provide a solution by applying Itô's Lemma to the calculation of the implied equity volatility from the asset value fluctuations. The implied equity risk is very much determined by the leverage ratio deployed:

$$\sigma_E = \sigma E_V \frac{V}{E}, \qquad (5)$$

where:

σ is the implied volatility of the company value.

σ_E is the implied equity volatility.

V is the company value at closing.

E is the equity value at closing.

E_V is the partial derivate of the equity value with respect to the company value.

The implied equity volatilities of the individual transactions are listed in column eight of table 2. The implied equity risks are quite large, ranging from 63.1% p.a. to 168.5% p.a. with a mean of 98.7% p.a. a median of 101.5% p.a. and a standard deviation of 22.7% p.a. Using the internal rates of return reported by the equity sponsors, it is now possible to calculate the Sharpe ratios (Sharpe 1966) for the individual transactions. The Sharpe ratios listed in the right hand column in table 2, range from -0.34 to 2.53, with a mean average of 0.72, a median of 0.59, and a large standard deviation of 0.66. The ratios could be used to benchmark the transactions with foregone investments for the same holding period and allowed performance analysis of individual transactions possible for the first time. However, since we have neither a sufficiently large or unbiased sample of LBO transactions we should not comprehensively compare our results e.g. with those of the above-mentioned literature. We merely highlight the magnitude of the implied equity volatility, which labels our sample transactions as very risky - on average riskier than the 86% p.a. volatility reported by Cochrane (2005).

Given the calculations of the implied volatilities and the Sharpe ratios for the LBO transactions, it is now relevant to analyse the sensitivity of the input parameters used in our approach. An increase in the asset value V, leaving all other parameters equal, should also lead to an increase in σ. The reason is that an increase in V would simultaneously raise the value of debt D. But, since the value of debt must remain the same, volatility has to increase to offset this effect. Similarly, if the value of D becomes larger, but V stays constant, volatility must decrease. A longer time to maturity lowers the value of debt and, in compensation, volatility should decrease. Similarly, if amortizations are paid later and t increases, the value of debt diminishes; this again must be balanced by a shrinking σ. Consequently, a potential decrease in the value of debt implied by a higher interest rates r or a larger coupon c will also be compensated for by lower volatility.

To confirm our expectations, we calculate variations in the implied asset value fluctuation brought by 1% changes *ceteris paribus* (respectively, percentage point changes) in the underlying variables. We also perform an analogous calculation; rather than fix the coupon at 8% for all transactions, we apply a constant 3% credit spread to the risk-free interest rate for all transactions. Results are described in table 3; as can be expected, we confirm the rationale of the model.

Insert table 3 here

Table 3 describes the percentage changes in asset value fluctuations determined by 1% increases *ceteris paribus* in the asset value (V), the value of debt (D), or the holding period (T). It also describes the changes if we delay *ceteris paribus* (t) the redemption payment to 51% of the holding period, and if we increase *ceteris paribus* the risk-free interest rate (r) or the debt coupon (c) by 1% point. The last column shows the new asset value volatility for the assumption that the debt coupon is not 8% p.a. in every transaction, but determined by a constant credit spread of 3% p.a. on the risk-free interest rate. The final rows show the most important descriptive statistics of the resulting asset volatility changes.

6. Summary and Conclusions

LBOs play an increasingly important role as a financing alternative in corporate lifecycles and as an asset class for institutional investors. However, literature does not come to a conclusion on a common way to benchmark these transactions from an investor's point of view. Risk measures so far derived by Peng (2001), Quigley and Woodward (2003), Ljungqvist and Richardson (2003), Cochrane (2005), and Groh and Gottschalg (2007) differ greatly in terms of scales, and either focus on systematic risk factors, or on value fluctuations of aggregated

portfolios of LBO transactions, and fail to take into account individual degrees of leverage. We use the CCA model (Ho and Singer 1984) to calculate implied idiosyncratic risks in LBO transactions. This model is very suitable for the valuation of debt in LBO transactions because it considers amortization. The asset value volatility and the equity value volatility can be derived from the model via a numerical procedure. For a sample of 40 LBO transactions we determine the necessary model parameters and calculate the implied asset and equity volatilities. We verify anticipated sensitivities in the model by varying the parameters. Having data on the returns to the equity investors of the transactions, we are able to calculate Sharpe Ratios for individual transactions for the first time, fully incorporating the superimposed leverage risks.

The underlying model can be criticized on a number of fronts. Despite the usual restrictions on this kind of CCA model, such as continuous trading, or flat and non-stochastic yield curves, the most important constraints in the proposed method are the assumptions of single layers of debt and equity without further option or conversion rights. Furthermore, a closed form solution such as that by Ho and Singer (1984) is only available under the simplified assumption of one single amortization payment.

However, we demonstrate that, with acceptable simplifications, the model can be applied to, and is suitable for measuring, the particular risks of LBO transactions. Due to a lack of available data, we are unable to perform comprehensive empirical research on risk-adjusted returns of the asset class with our proposed method. Nonetheless, our method can be used for benchmarking current and future transactions once more and accurate data for the calculation of necessary model parameters becomes available.

Table 1: Descriptive Statistics of Sample Data

	Min	Max	Average	Median	Stdandard-deviation.
Closing Date	Apr 87	Mar 03	Oct 96	Jun 97	45.2 months
Exit Date	Nov 93	Dec 03	Feb 00	Jun 99	38.1 months
Holding Period [years]	0.25	9.92	3.32	2.42	2.32
Enterprise Value at Closing [$m]	3.6	1,679.7	181.7	95.9	290.9
Enterprise Value at Exit [$m]	10.5	1,347.5	227.5	131.5	263.9
Initial Debt/Equity	0.3	8.0	2.8	2.5	1.9
Exit Debt/Equity	0	3.1	0.7	0.4	0.7
Equity Investment [$m]	0.4	248.4	47.8	25.0	56.0
Equity Payoff [$m]	8.0	631.2	140.8	95.2	139.6
IRR (p.a.)	-39.6%	267.5%	77.6%	57.9%	71.8%

Table 1 describes the most important parameters of our sample transactions.

Table 2: Idiosyncratic Risks of Our Sample Transactions

Transaction #	Holding Period [years]	Risk Free Interest Rate	Enterprise Value at Closing [$m]	Debt at Closing [$m]	Debt at Exit [$m]	Idiosyncratic Transaction Risk [p.a.]	Idiosyncratic Equity Risk [p.a.]	Reported IRR [p.a.]	Sharpe-Ratio
1	5.50	5.16%	21.3	15.3	0.0	**24.70%**	**69.66%**	75.3%	**1.01**
2	6.33	4.96%	95.0	72.1	57.8	**26.01%**	**75.38%**	188.0%	**2.43**
3	0.25	5.15%	58.5	44.0	42.6	**44.05%**	**168.46%**	-22.6%	**-0.16**
4	1.33	5.08%	67.0	48.0	30.8	**37.48%**	**115.48%**	193.2%	**1.63**
5	2.00	4.37%	364.6	260.7	185.8	**36.87%**	**106.75%**	104.8%	**0.94**
6	3.92	5.55%	35.5	19.7	17.7	**38.87%**	**77.17%**	56.0%	**0.65**
7	7.50	5.65%	312.4	272.4	262.0	**16.70%**	**73.91%**	33.7%	**0.38**
8	5.58	4.01%	315.0	243.0	170.6	**28.36%**	**83.33%**	23.7%	**0.24**
9	1.00	5.52%	17.0	11.0	11.0	**39.84%**	**104.35%**	130.9%	**1.20**

10	4.17	5.36%	117.5	87.5	77.9	**27.23%**	**81.47%**	49.8%	**0.55**
11	5.08	4.83%	10.0	5.7	2.4	**38.39%**	**76.29%**	29.0%	**0.32**
12	3.58	5.00%	6.4	3.1	1.3	**47.63%**	**84.12%**	55.0%	**0.59**
13	9.92	4.95%	45.0	35.4	20.6	**22.16%**	**65.83%**	23.5%	**0.28**
14	2.50	2.44%	211.0	107.6	30.0	**57.61%**	**104.82%**	25.0%	**0.22**
15	3.92	4.62%	16.9	11.9	7.1	**32.36%**	**85.94%**	55.7%	**0.59**
16	4.83	4.42%	3.6	3.2	2.5	**19.28%**	**93.67%**	80.4%	**0.81**
17	2.17	4.50%	164.4	139.4	30.4	**21.45%**	**104.74%**	205.5%	**1.92**
18	1.17	5.25%	116.5	83.0	75.3	**36.66%**	**112.85%**	135.8%	**1.16**
19	2.08	5.33%	38.0	30.5	19.7	**25.66%**	**102.85%**	266.0%	**2.53**
20	1.42	1.78%	132.0	74.7	14.3	**59.80%**	**123.94%**	67.3%	**0.53**
21	0.75	1.24%	74.9	25.9	20.6	**103.23%**	**152.27%**	166.4%	**1.08**
22	4.67	3.74%	51.3	25.2	0.0	**45.50%**	**79.60%**	21.0%	**0.22**
23	6.58	6.46%	234.0	167.0	150.5	**22.61%**	**63.11%**	1.8%	**-0.07**
24	1.92	2.78%	88.0	17.8	0.0	**100.77%**	**123.85%**	-39.6%	**-0.34**
25	2.33	4.67%	56.0	41.4	0.0	**28.56%**	**92.42%**	144.2%	**1.51**
26	1.75	5.58%	125.5	85.2	57.4	**36.82%**	**101.40%**	68.0%	**0.62**
27	2.50	2.44%	350.0	188.5	154.2	**55.27%**	**103.93%**	30.0%	**0.27**
28	9.17	5.00%	323.0	248.0	133.9	**23.12%**	**66.30%**	14.2%	**0.14**
29	5.25	4.31%	30.0	20.7	13.4	**32.93%**	**80.66%**	40.7%	**0.45**
30	4.00	4.16%	63.7	44.9	41.5	**33.84%**	**87.26%**	27.9%	**0.27**
31	1.33	5.57%	1,679.7	1,431.3	818.0	**21.82%**	**118.64%**	79.2%	**0.62**
32	4.50	3.83%	40.9	24.6	21.0	**41.38%**	**84.54%**	11.4%	**0.09**
33	1.33	5.51%	75.0	54.4	54.2	**31.91%**	**102.08%**	45.3%	**0.39**
34	1.08	5.17%	417.3	243.6	209.6	**50.57%**	**113.22%**	77.3%	**0.64**
35	2.75	5.44%	96.8	81.8	59.5	**20.96%**	**97.48%**	59.8%	**0.56**
36	2.00	2.74%	103.0	51.2	23.7	**62.53%**	**112.52%**	26.0%	**0.21**
37	1.25	5.47%	172.2	152.0	138.9	**19.90%**	**128.04%**	267.5%	**2.05**
38	1.67	5.41%	857.3	677.0	370.0	**27.20%**	**107.18%**	108.1%	**0.96**
39	1.58	5.44%	100.0	75.0	64.7	**31.36%**	**106.45%**	75.3%	**0.66**
40	2.08	2.68%	182.1	133.4	76.8	**39.39%**	**114.86%**	103.4%	**0.88**
					Minimum	**16.70%**	**63.11%**	-39.6%	**-0.34**
					Mean	**37.77%**	**98.67%**	77.6%	**0.72**
					Median	**33.39%**	**101.74%**	57.9%	**0.59**
					Maximum	**103.23%**	**168.46%**	267.5%	**2.53**
					Standard Deviation	**18.97%**	**22.72%**	71.8%	**0.66**

The six columns on the left of table 2 list the model parameters necessary to calculate the implied transaction risks and the implied equity risks for our 40 sample transactions. Columns seven and eight depict the resulting risks. Column nine shows the internal rates of return reported by the transaction equity sponsors, while column ten shows the Sharpe Ratios. The final rows of the table show the most important descriptive statistics for our results.

Table 3: Sensitivity Analysis

Transaction #	V	D	T	t	r	c	3% Credit Spread
1	0.43%	-0.44%	-0.52%	-0.08%	-3.30%	-3.26%	24.21%
2	0.46%	-0.48%	-0.38%	-0.19%	-3.68%	-1.05%	26.05%
3	1.01%	-0.28%	-0.40%	-0.42%	-3.69%	-0.13%	44.03%
4	0.72%	-0.74%	-0.44%	-0.31%	-3.95%	-0.95%	37.40%
5	0.62%	-0.63%	-0.38%	-0.31%	-3.39%	-0.76%	37.35%
6	0.36%	-0.31%	-0.51%	-0.33%	-4.92%	-0.99%	38.33%
7	0.54%	-0.58%	-0.34%	-0.13%	-3.70%	-0.79%	16.18%
8	0.51%	-0.55%	-0.34%	-0.20%	-3.12%	-0.98%	29.33%
9	0.56%	-0.19%	-0.44%	-0.37%	-4.53%	-0.26%	39.70%
10	0.47%	-0.47%	-0.39%	-0.23%	-3.95%	-0.76%	26.95%
11	0.39%	-0.42%	-0.54%	-0.25%	-4.15%	-2.13%	38.73%
12	0.40%	-0.42%	-0.62%	-0.34%	-4.69%	-2.25%	47.63%
13	0.43%	-0.46%	-0.41%	-0.12%	-3.49%	-1.73%	22.24%
14	0.51%	-0.54%	-0.50%	-0.37%	-3.13%	-1.86%	61.97%
15	0.50%	-0.54%	-0.41%	-0.24%	-3.61%	-1.25%	32.83%
16	0.71%	-0.76%	-0.26%	-0.15%	-2.79%	-0.59%	19.64%
17	0.76%	-0.78%	-0.33%	-0.11%	-2.43%	-1.60%	22.20%
18	0.63%	-0.47%	-0.39%	-0.34%	-3.85%	-0.40%	36.56%
19	0.70%	-0.73%	-0.37%	-0.20%	-3.61%	-0.91%	25.36%
20	0.67%	-0.70%	-0.49%	-0.36%	-2.74%	-1.89%	65.23%
21	0.56%	-0.24%	-0.60%	-0.92%	-3.37%	-0.55%	105.27%
22	0.38%	-0.38%	-0.59%	-0.19%	-3.32%	-3.30%	49.14%
23	0.37%	-0.39%	-0.54%	-0.18%	-6.13%	-1.45%	20.47%
24	0.42%	-0.42%	-0.90%	-0.61%	-4.02%	-3.97%	108.20%
25	0.60%	-0.60%	-0.47%	-0.11%	-2.85%	-2.84%	29.40%
26	0.60%	-0.61%	-0.49%	-0.31%	-4.65%	-1.07%	36.20%
27	0.46%	-0.38%	-0.40%	-0.47%	-3.02%	-0.66%	56.98%
28	0.42%	-0.45%	-0.42%	-0.12%	-3.57%	-1.82%	23.12%
29	0.45%	-0.48%	-0.40%	-0.23%	-3.50%	-1.23%	33.76%
30	0.47%	-0.44%	-0.34%	-0.28%	-3.25%	-0.63%	34.38%
31	0.86%	-0.90%	-0.35%	-0.17%	-3.30%	-0.93%	21.29%
32	0.40%	-0.38%	-0.40%	-0.33%	-3.41%	-0.85%	42.39%
33	0.57%	-0.33%	-0.37%	-0.30%	-3.96%	-0.30%	31.76%
34	0.56%	-0.36%	-0.50%	-0.46%	-4.54%	-0.56%	50.48%
35	0.69%	-0.73%	-0.33%	-0.17%	-3.46%	-0.74%	20.64%
36	0.53%	-0.55%	-0.50%	-0.47%	-3.35%	-1.38%	65.47%
37	0.87%	-0.87%	-0.26%	-0.18%	-2.96%	-0.24%	19.78%
38	0.73%	-0.76%	-0.40%	-0.21%	-3.68%	-1.16%	26.73%
39	0.63%	-0.57%	-0.38%	-0.28%	-3.89%	-0.50%	31.14%
40	0.70%	-0.74%	-0.33%	-0.30%	-2.70%	-0.84%	41.29%
Minimum	0.36%	-0.90%	-0.90%	-0.92%	-6.13%	-3.97%	16.18%
Mean	0.57%	-0.53%	-0.44%	-0.28%	-3.64%	-1.24%	38.50%
Median	0.55%	-0.48%	-0.40%	-0.27%	-3.53%	-0.96%	34.07%
Maximum	1.01%	-0.19%	-0.26%	-0.08%	-2.43%	-0.13%	108.20%
Standard Deviation	0.15%	0.17%	0.12%	0.16%	0.70%	0.89%	20.32%

Table 3 describes the percentage changes in asset value fluctuations determined by 1% increases ceteris paribus in the asset value (V), the value of Debt at closing (D), or the holding period (T). It also describes the changes if we postpone, ceteris paribus, the redemption payment to 51% (t is increased) of the holding period, and if we increase, ceteris paribus, the risk-free interest rate (r), or the debt coupon (c) by 1% point. The last column shows the new asset value volatility for the assumption that the debt coupon is not 8% p.a. in every transaction, but determined by a constant credit spread of 3% points on the risk-free rate. The final rows show the most important descriptive statistics of the resulting asset value volatility changes.

Appendix

Brent's algorithm (Brent 1973) is an efficient way to numerically solve for the root of an equation without calculating the derivative. It basically applies quadratic interpolation based on three points, but switches to bisection if this interpolation does not lead to an improvement.

Let f denote the target function. In our case f is the value of debt given in equation (1) minus the target debt value to be matched. The algorithm requires two starting points σ_0, σ_1 such that $f(\sigma_0)$ and $f(\sigma_1)$ have different signs. It is $\sigma_2 = \sigma_1$ defined.

A new value σ^* is calculated as follows:

If $f(\sigma_0) <> f(\sigma_2)$ and $f(\sigma_1) <> f(\sigma_2)$, then

$$\sigma^* = \frac{\sigma_0 f(\sigma_1) f(\sigma_2)}{(f(\sigma_0) - f(\sigma_1))(f(\sigma_0) - f(\sigma_2))}$$
$$+ \frac{\sigma_1 f(\sigma_0) f(\sigma_2)}{(f(\sigma_1) - f(\sigma_0))(f(\sigma_1) - f(\sigma_2))} \quad ,$$
$$+ \frac{\sigma_2 f(\sigma_0) f(\sigma_1)}{(f(\sigma_2) - f(\sigma_0))(f(\sigma_2) - f(\sigma_1))}$$

otherwise

$$\sigma^* = \sigma_1 - \frac{f(\sigma_1)(\sigma_1 - \sigma_0)}{f(\sigma_1) - f(\sigma_0)} .$$

This new σ^* is only adapted if

$$\left(\sigma^* - \frac{3\sigma_0 + \sigma_1}{4} \right)(\sigma_1 - \sigma^*) \leq 0 ,$$

otherwise

$$\sigma^* = \frac{\sigma_0 + \sigma_1}{2} .$$

With this new value the three node points are updated:

$\sigma_2 = \sigma_1$

If $f(\sigma_0) \cdot f(\sigma^*) < 0$ then

$\sigma_1 = \sigma^*$,

otherwise

$\sigma_0 = \sigma^*$.

If $| f(\sigma_0) | < | f(\sigma_1) |$ then σ_0 and σ_1 are switched.

The algorithm stops if $| \sigma_0 - \sigma_0 |$ is smaller than a predefined accuracy, in our case 0.01%.

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